JOURNAL OF SPACECRAFT AND ROCKETS
Vol. 29, No. 5, September-October 1992

Aerodynamic Design of Axisymmetric Hypersonic Wind-Tunnel Nozzles Using a Least-Squares/Parabolized Navier-Stokes Procedure

John J. Korte*
NASA Langley Research Center, Hampton, Virginia 23665

A new procedure, which unifies the best of current classical design practices, computational fluid dynamics (CFD), and optimization procedures, is demonstrated for designing the aerodynamic lines of hypersonic wind-tunnel nozzles. The new procedure can be used to design wind-tunnel nozzles with thick boundary layers where the classical design procedure has been shown to break down. An efficient CFD code, which solves the parabolized Navier-Stokes (PNS) equations, is coupled to a least-squares (LS) optimization procedure. An LS problem is formulated to minimize the difference between the computed flowfield and the objective function, consisting of the centerline Mach number distribution and the exit Mach number and flow angle profiles. The aerodynamic lines of the nozzle are defined using a cubic spline, the slopes of which are optimized with the design procedure. The advantages of the new procedure are that it allows full use of CFD codes in the design process, it can be used to design new nozzles or improve sections of existing nozzles, and it automatically compensates the nozzle contour for viscous effects. The computed flowfield for a Mach 15 helium LS/PNS designed nozzle is compared with the classically designed nozzle and demonstrates a significant improvement in the flow expansion process and uniform core region.

Nomenclature

a_i	= coefficients of cubic spline
f	= least-squares error vector
f_i	= component of f
I	= index of f
i	= grid index, axial direction
$imax$	= total grid points, axial direction
J	= Jacobian matrix
j	= grid index, radial direction
k	= least-squares iteration index
M_{axis}	= design axis Mach number
M_{design}	= test section design Mach number
M_x	= computed axial Mach number
m	= number of components in f
N	= total knots on the cubic spline
n	= number of components in X
Obj	= objective function
r_t	= original throat radius
X	= design parameter vector
X_i	= component of X
x	= axial coordinate
x_{1-N}	= axial coordinate of spline knots
y	= radial coordinate
y_w	= nozzle wall radius
θ	= flow angle, rad
$\omega_M, \omega_{MA}, \omega_\theta$	= weighting factors

Introduction

AXISYMMETRIC wind tunnels have been used for the last 30–40 years for simulating the hypersonic flight environment. However, the flow quality of many of these nozzles is not adequate for current research requirements, which now include validating computational fluid dynamic (CFD) computer codes. Efforts have been made to build and update existing hypersonic facilities with new and improved nozzle aerodynamic lines.[1,2] The state of the art in the aerodynamic design of wind-tunnel nozzles is still based on the procedure proposed by Prandtl and Buseman[3] in 1929. This classical procedure requires that the inviscid contour be designed using the method of characteristics (MOC) and corrected with a displacement thickness obtained from a boundary-layer (BL) solution. Recent computational analysis[4] of hypersonic wind-tunnel nozzles designed with the method of characteristic/boundary-layer (MOC/BL) procedure, using solutions to the Navier-Stokes (NS) equations, has shown severe limitations in obtaining the desired flow quality and test conditions when the boundary layer becomes a large percentage of the nozzle's diameter. The MOC/BL design procedure assumes that the characteristics reflect at the location of the inviscid contour. Candler and Perkins[5] showed that, for large boundary layers, typical of hypersonic wind-tunnel nozzles, the characteristics reflect between the wall and the location of the inviscid contour. Therefore, the actual characteristics of the nozzle lag the characteristics used in the design procedure. In thick boundary layers the actual characteristics are no longer properly canceled, and the flow quality of the nozzle deteriorates. In this paper a design procedure that automatically avoids this effect will be presented.

The classical theory used in the design procedure of supersonic wind tunnels can be found in a number of reports[6-10] and books.[11-13] The basic assumption made is that the boundary layer is small, compared to the characteristic length (nozzle radius), so that the nozzle flowfield can be treated as inviscid for designing the aerodynamic lines. Once the aerodynamic lines are determined, a correction is made to account for the displacement thickness of the boundary layer. This basic procedure has been applied successfully to both supersonic and low Mach number hypersonic nozzles.

Presented as Paper 92-0332 at the AIAA 30th Aerospace Sciences Meeting, Reno, NV, Jan. 6-9, 1992; received Jan. 11, 1992; revision received March 25, 1992; accepted for publication March 25, 1992.

*Research Scientist, Theoretical Flow Physics Branch, Fluid Mechanics Division. Senior Member AIAA.

The inviscid design procedure requires the position of the sonic line at the throat, a prescribed centerline Mach number or pressure distribution, and the condition of uniform flow at the nozzle exit. Once a centerline Mach number distribution has been specified, the MOC can be used to determine an inviscid contour. This inviscid contour can be scaled to any particular size. The design contour for one operating condition is determined by computing the displacement thickness using a boundary-layer code and adding it to the inviscid contour. Modern implementations of the MOC/BL design procedure usually differ only on how the sonic line and centerline Mach number distribution are specified and which numerical technique is used to compute the inviscid flowfield and the boundary-layer solution.

Recent designs of hypersonic wind-tunnel nozzles made with the MOC/BL procedure have been evaluated using CFD codes solving the NS equations.[2,4,5,14] Attempts to improve the resulting nozzle design using the CFD codes are difficult because of the computational time required to solve the NS equations and the trial and error process of changing the contour. A mathematical procedure for correcting an existing contour is given by Meyer and Holt,[15] but no application of this procedure to a wind-tunnel nozzle design has been documented. For modifying existing wind-tunnel complexes, special design procedures may also be necessary for the newly designed nozzle to mate with existing components. A special CFD procedure[1] was recently developed to design a Mach 4 and a Mach 5 nozzle for the NASA Langley 8-Foot High Temperature Tunnel. The new nozzles were designed in reverse, so that they would match with the exit portion of the existing Mach 7 nozzle.

From the preceding discussion, it is clear that a new procedure is needed for designing hypersonic nozzles that will produce high-quality, uniform flowfields in the test section. To design the hypersonic nozzles for a uniform flowfield accurately, the procedure must include the effects of the thick boundary layer on the inviscid flowfield. In addition, it may be necessary to include the effects of intermolecular forces, vibrational, and chemical nonequilibrium on the test gas. These effects will be included if the design procedure is based on the solution of the parabolized Navier-Stokes (PNS) equations with the appropriate thermodynamic/chemical model for the gas.

The PNS equations are a subset of the NS equations, which can be integrated using efficient space marching procedures. These equations are obtained from the NS equations by neglecting streamwise diffusion effects and a portion of the subsonic streamwise pressure gradient. For many practical flowfields, such as hypersonic nozzles, these approximations are valid as long as streamwise separation does not occur. To model the curvature of the nozzle contour accurately, a large number of streamwise stations need to be computed, especially in the throat region. Previously, the cost associated with using even a PNS solver in a design procedure would have been prohibitive. With the increased use of supercomputers, efficient and fast CFD solvers have been developed for the PNS equations.[16,17] The explicit upwind noniterative algorithm for solving the PNS equations of Ref. 17 has been modified to solve internal axisymmetric flowfields. A typical Mach 15 nozzle 130 in. long can be simulated using over 90,000 streamwise stations in under 90 s of Cray 2 CPU time. A typical Mach 6 nozzle can be computed in 10 s of Cray 2 CPU time. The use of this solver for the PNS equations has made it practical to consider implementation of an iterative CFD-based design procedure.

The use of CFD codes in design has usually been to provide a direct analysis of the performance of existing or proposed designs. Coupling CFD codes with design procedures has been done for a number of limited applications. Most of these applications have been used to improve a surface pressure distribution over a wing or other simple surface. However, these procedures were not general enough to handle a design that would produce a desired flowfield away from the surface. Recently, Huddleston[18] developed a CFD optimization method that is flexible enough to be used for designing nozzles and other flow components. Huddleston's optimization method is based on minimizing an objective function by solving a nonlinear least-squares (LS) problem that will result in an optimal solution for a parametric description of a flow surface. He has applied this method to both two- and three-dimensional nozzles and airfoils, coupled with both Euler and NS flow solvers. However, he did not apply the method to a flowfield that included a thick boundary-layer region or a PNS flow solver. The disadvantage of this method is the expense associated with computing sensitivity derivatives necessary to solve the LS minimization problem. The advantage of the method is that it is not limited to a particular configuration or objective function.

In a previous paper the LS optimization procedure of Huddleston was coupled to an efficient PNS solver and used for optimizing existing hypersonic wind-tunnel contours.[19] In the LS/PNS procedure the aerodynamic contour was modeled by use of a cubic spline. The design parameters were the radii at fixed locations along the nozzle length. The objective function was the error of Mach number and radial velocity in the inviscid region of the nozzle exit. It was necessary to define the initial value of the radii by matching as closely as possible the original MOC/BL designed contour. Newton's method was then used to solve the LS problem. The results demonstrated that an improved exit plane profile could be obtained. However, it was difficult to obtain good initial values of the design parameters and to reach a strong minimum of the objective function. Additional work is needed to improve the method so that it can also be applied for designing wind-tunnel nozzles with specified physical characteristics.

In this paper, the LS/PNS procedure is extended for use in designing hypersonic wind-tunnel nozzles. A new form of the objective function is given that defines the shape of the nozzle by specifying the centerline Mach number distribution and the uniform flow exit condition. The design parameters are modified to be the coefficients of the spline associated with the local nozzle wall slope. The motivation for these two modifications resulted from an attempt to be consistent with the constraints and design methodology used in the MOC/BL design procedures. These two modifications greatly improved the convergence of the optimization problem. The new procedure is demonstrated by designing four helium nozzles. The computed flowfield for the Mach 15 LS/PNS designed nozzles is then compared with a MOC/BL designed nozzle. The robustness of the procedure is demonstrated by designing a Mach 12 and a Mach 18 nozzle using the same starting contour as that used in the Mach 15 design.

Aerodynamic Design Procedure for Hypersonic Nozzles

Assume that the shape of the subsonic and transonic contours of the nozzle have been specified and that an NS solution exists for these sections. An LS/PNS optimization procedure[19] is extended for designing the supersonic/hypersonic portion of the nozzle. The details of the PNS solver can be found in Ref. 17. In this section a brief review of the LS/PNS optimization procedure used in Ref. 19 will be presented, and the changes developed for the application to the design of hypersonic nozzles will be emphasized.

Objective Function

A nonlinear optimization problem is solved for a set of design parameters by minimization of an objective function. Consider the minimization of an objective function Obj that is dependent on a set of design parameters X and is constructed from a series of functions f_I of the nonlinear LS form[20]

$$Obj(X) = \sum_{I=1}^{m} f_I^2(X) \tag{1}$$

It is convenient to define the vectors f and X as

$$f = [f_1, f_2, \ldots, f_m]^T \tag{2}$$

$$X = [X_1, X_2, \ldots, X_n]^T \tag{3}$$

so that the objective function can be written as

$$Obj(X) = f^T(X) f(X) \tag{4}$$

For each particular minimization problem, there are a number of different possibilities for the definition of f. It is desirable that f be selected so that Obj has a strong minimum and satisfies the design criteria. Recall that the MOC design procedure requires the centerline Mach number M_{axis} and the condition of parallel flow at the design Mach number M_{design} at the nozzle exit. The same requirements are used here to form an objective function. The components of f at the nozzle exit associated with the error in axial Mach number M_x and flow angle θ are

$$f_I = \omega_M (M_{x_{imax,j}} - M_{\text{design}}) \tag{5}$$

$$f_{I+1} = \omega_\theta (\theta_{imax,j} - 0) \tag{6}$$

and along the centerline

$$f_{I+2} = \omega_{MA} (M_{x_{i,1}} - M_{\text{axis}}) \tag{7}$$

where ω_M, ω_{MA}, and ω_θ are weighting functions used to adjust the relative tolerances between the different types of error terms. The $imax, j$ index identifies one of the grid points used to evaluate f at the exit of the nozzle and the i, 1 index a point on the nozzle centerline.

The selection of a centerline Mach number distribution determines the overall length of the nozzle and the contour for the initial expansion, which is somewhat arbitrary. However, from a practical viewpoint, the initial expansion should smoothly blend into the turning contour. If this is done, it is more likely that the contours can be accurately machined. Sivells[8] specified a Mach number distribution that matches theoretical transonic conditions at the throat, conical-source flow conditions through an intermediate region, and design flow conditions at the nozzle exit. The advantage of Sivells's centerline distribution is that it smoothly matches the different flow regions, eliminating discontinuous changes in contour curvature. For the new design procedure, the centerline Mach number distribution is specified using the method proposed by Sivells. In Sivells' method the centerline distribution consisted of four sections: 1) a fourth-order polynomial between the transonic section and the conical-source flow regime, 2) a source-flow region, 3) a fifth-order polynomial between the source-flow region and the parallel flow regime, and 4) the parallel flow region at the design Mach number (Fig. 1). The coefficients of the polynomials are determined by matching the Mach number and its derivatives with respect to nozzle length, at the beginning and end of each section. Sivells gave equations for the derivatives of the source-flow regime, the derivatives at the nozzle sonic line using a transonic series solution, nozzle length, and expressions for determining the transition Mach number where the different sections should start. Unfortunately, these relationships are limited to an ideal gas. Sivells' equations are used for defining the centerline Mach number distribution since our current nozzle designs are for helium, which can be accurately modeled as an ideal gas over the temperature and pressure ranges used herein. However, since an NS solution is computed for the transonic section, these derivatives could be calculated directly from the numerical results. The derivatives necessary for the source-flow region could also be easily computed directly, using the same gas model as that used in the CFD code.

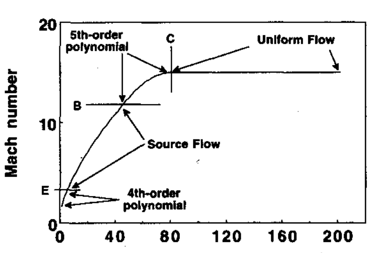

Fig. 1 Design centerline Mach number distribution.

Design Parameters

Like the objective function, the selection of the design parameters depends on the problem being solved. The ideal set would contain the minimum number of elements and be strongly coupled to the objective function. The design parameters are usually coefficients used to define a wall boundary or quantities that define the flowfield conditions. For the design of a nozzle the flow conditions are usually given, and the wall contour needs to be determined.

Wall Contour

For a contoured nozzle there is no obvious choice of a function to describe the geometry that would minimize the number of design parameters. A contoured nozzle can have an arbitrary expansion up to the inflection point. After the inflection point the nozzle contour is determined so that the proper wave cancellation occurs and parallel flow is obtained at the exit of the nozzle. Many contoured nozzles have a curvature discontinuity at the inflection point, i.e., the second derivative is not continuous. In Ref. 19 a cubic spline with five knots was used to model the contoured nozzle. The total number of knots on the spline and the design parameters necessary for defining the contoured nozzle depend on how accurately the design problem is to be determined. The selection of the number of knots and design parameters will be discussed later in this section. The nozzle contour is specified using N cubic polynomial equations:

$$0 < x \le x_1, \qquad \{r = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

$$x_1 < x \le x^2, \qquad \{r = a_4 + a_5 x + a_6 x^2 + a_7 x^3$$

$$\vdots$$

$$x_{N-1} < x \le x_N$$

$$\{r = a_{4N-4} + a_{4N-3} x + a_{4N-2} x^2 + a_{4N-1} x^3 \tag{8}$$

The N cubic equations result in $4N$ coefficients, assuming that the locations x_1 through x_N are known. The requirement of continuity of the surface, slope, and curvature at x_1 through x_N specifies $3(N-1)$ of the coefficients. The remaining $N+3$ coefficients are specified by using the inlet and exit radii and slopes and by specifying the slope at $N-1$ locations. Once the aforementioned constants are defined, a linear system of equations are solved to determine the value of the coefficients $a_0 - a_{4N-1}$.

Selection of Design Parameters

In Ref. 19, selected radii at locations x_1 through x_N were used as design parameters. Recall that in the MOC procedure the wall is determined by the slope necessary to turn the flow in the correct direction. The use of wall slopes as design parameters instead of radii is more consistent with the physics of supersonic flowfields. The adoption of a slope-based design parameter greatly improved the convergence of the design procedure. The design parameters used are the coefficients in Eq. (8) for a_5, a_9, . . ., a_{4N-3}, which represent the slopes at points x_1, x_2, . . ., x_{N-1}. In addition to these, other control parameters were used, depending on the constraints necessary for a particular nozzle design. Typical constraints may be fixed length, nozzle throat, or nozzle exit.

Number of Knots on the Cubic Spline

Determining the number of knots on the cubic spline and the location of the control points x_1 through x_N is an ad hoc process. The procedure adopted is that a preliminary design is made using the predetermined centerline Mach number distribution and then knots and design parameters are added until a converged solution is obtained that satisfies the design requirements.

Nonlinear Optimization by Solving an LS Problem

Now that an objective function and the design parameters have been defined, the problem of minimizing Eq. (4) has to be addressed. Recall that $Obj(X)$ will have a minimum value when its gradient is zero. Newton's method for nonlinear least squares can be written directly as

$$J_k^T J_k \Delta X_k = -J_k^T f_k \tag{9}$$

where the design parameters are updated by

$$X_{k+1} = X_k + \Delta X_k \tag{10}$$

This is the small residual problem form of the LS problem. For additional information on methods for solving Eq. (9) for both small and large residual problems, see the text by Scales.[20]

Sensitivity Derivatives

The sensitivity derivatives are the elements of the Jacobian matrix J. The calculation of the sensitivity derivatives is the most expensive part of the optimization procedure. Each element of the Jacobian matrix requires a separate PNS flowfield solution. The elements of the Jacobian matrix are approximated using forward finite differences:

$$\frac{\partial f_I}{\partial X_J} = \frac{f_I(X + \Delta X_J) - f_I(X)}{\Delta X_J} \tag{11}$$

The accuracy of the difference approximation is dependent on both the size of ΔX_J and the flowfield solution. The Jacobian elements in this study were computed by specifying ΔX_J to be $X_J * 10^{-6}$.

Updating the Jacobian Matrix

The expensive portion of the preceding algorithm is the determination of the elements of the Jacobian matrix. One evaluation of the objective function $Obj(X)$ is needed for each element of X. If X contains n elements, then each iteration requires $n + 1$ functional evaluations or flowfield solutions. An alternate method to numerically computing the Jacobian matrix would be to use an analytic update formula such as Broyden's.[20] Previously, our attempts to use Broyden's update formula when the design parameters were the radii were unsuccessful.[19] However, after we switched to the slope-based design parameters, we successfully used Broyden's update formula.

Application of the LS/PNS Design Procedure

To demonstrate the new procedure, three helium nozzle aerolines are designed. The nozzles are required to have an exit radius of 10.44 in. and a total length of 125 in. The design contour is to be computed for a stagnation flow condition of 1500 psi and 530°R. The wall temperature can be assumed to be constant at 530°R, and the flow can be assumed to be turbulent. The design exit conditions for the Mach 15 flowfield are a 47% core flow, with a goal of less than ±2% variation in static pressure and less than 0.287 deg variation in flow angle (0.5% of axial velocity). This particular nozzle was selected because a MOC/BL design had been made for Mach 15 that failed to meet these design requirements (when evaluated with a NS code). The same starting contour will then be used to design a Mach 12 and a Mach 18 nozzle using the same constraints to demonstrate the robustness of the method.

The hypersonic portion of the nozzle flowfield is computed by use of an axisymmetric version of the explicit upwind PNS solver of Ref. 17. The subsonic and transonic flowfield of the nozzle is computed only once with the Navier-Stokes equations using an axisymmetric version of NASCRIN[21]. The flowfield is assumed to be turbulent and is simulated by use of the Baldwin and Lomax[22] eddy viscosity model. The radial direction is discretized using 50 points, with point clustering at both the wall and the nozzle axis. The total number of streamwise stations was ~90,000. The computer runs were made on Langley's Cray 2 supercomputer.

The first iteration of the design procedure is started by forming the elements of the Jacobian matrix numerically and then solving the nonlinear LS problem. For the second iteration the elements are updated with Broyden's method. Broyden's update procedure is used until there is no longer any improvement in the convergence of the objective function toward a minimum. The elements are then recomputed numerically, and the procedure continues until either the desired design performance or a minimum is obtained.

Design Parameters

The aerodynamic lines are modeled using 15 knots on a cubic spline. The initial values of the spline coefficients are obtained from evaluating the slope at the selected locations of the MOC/BL designed nozzle contour and are shown in Table 1. The number and location of the knots were selected so that changes in the slope of the aerodynamic lines would be accurately represented (Fig. 2). The initial values do not have to come from a MOC/BL design, but should represent a shape similar to a nozzle. This is demonstrated when the initial values obtained for a Mach 15 design are used to design a Mach 12 and Mach 18 nozzle. A total of 15 design parameters are used in the optimization procedure. The first design parameter is the initial throat radius $r_t = 0.61324$ in. The wall slopes dy_w/dx at indexes 2–15 are the next 14 design parameters. The locations of the knots are held constant.

Table 1 Initial values of cubic spline slopes for helium nozzle

Index	x, in.	dy_w/dx
1	0.00000	0.00000
2	0.57757	0.08062
3	1.17757	0.13214
4	1.67757	0.16226
5	2.47757	0.18155
6	3.57757	0.19470
7	7.87757	0.19810
8	10.0777	0.18444
9	13.0776	0.16740
10	19.0776	0.14000
11	31.0776	0.10530
12	55.0776	0.06850
13	80.0776	0.04810
14	102.0776	0.03590
15	125.6600	0.02683

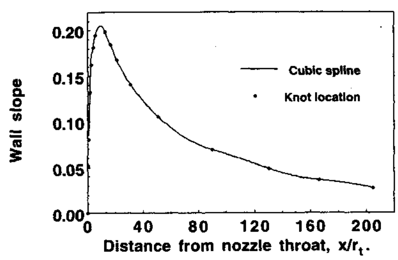

Fig. 2 Wall slope computed using a cubic spline.

Objective Function

The objective function consists of an array of 148 elements. The first 100 elements are the centerline Mach number errors, evenly spaced over the nozzle length; the next 24 elements are the exit core Mach number profile errors; and the final 24 elements are the exit core flow angle errors. The 24 elements used in the exit core are from the nozzle centerline to halfway across the exit radius, since a 47% core flow is desired. The weighting factors used are

$$\omega_M = 1.0/M_{\text{design}}, \qquad \omega_{MA} = 1.0/M_{\text{axis}(x)}, \qquad \omega_\theta = 1.0/0.2$$

The centerline Mach number distribution is computed using Sivells' formula by fixing the Mach number at points E, B, and C to be 2.3, 12, and 15, respectively (see Fig. 1) for the Mach 15 design cases. The source-flow region is computed for a conical half-angle of 12 deg. The ratio of the radius of curvature to the nozzle radius at the throat is 9 (used in computing the transonic Mach number distribution). The equations for computing the centerline distribution and recommendations for values of the coefficients can be found in Sivells' report.[8] This centerline distribution will result in a uniform flow region of ~9.8 in. (47% of nozzle exit diameter) at the nozzle exit. The length of the uniform flow region was multiplied by the tangent of the Mach angle to estimate the uniform flow region. Note that this is a cutoff contour, in that the nozzle length is less than it would be if the inviscid uniform flow region had been fully expanded to the design Mach number. This is common practice in hypersonic nozzles, where so much of the profile is part of the boundary layer.[11]

Design Logic

Since the significant viscous effects occur in the supersonic/hypersonic section of the nozzle, an approximate procedure is used at the throat. When the throat radius is a design parameter, the initial Navier-Stokes solution is scaled to a new throat radius after each iteration. Once the design parameters have converged, a new Navier-Stokes solution is computed in the upstream subsonic and transonic section. Since the boundary layer is extremely thin at the throat, one would expect only small differences between the approximate procedure and the new NS solution for the subsonic and transonic section in the inviscid core. The accuracy of this approximation will be investigated in the future by rerunning the subsonic and transonic section with the newly designed contour and recomputing the supersonic/hypersonic region with the new initial conditions.

Design for a Fixed Length and Exit Radius

The nozzle throat and the slope at 14 points are used as the design parameters. The objective function for the Mach 15 design is reduced two orders of magnitude (Fig. 3) from the initial guess. The change in the contour over the MOC/BL design is hardly noticeable (Fig. 4). To compare differences in nozzle performance, a CFD solution was computed for the MOC/BL designed nozzle using the same CFD codes and subsonic/transonic solution to start the PNS calculation. However, significant differences are demonstrated between their flowfield solutions. The centerline Mach number almost

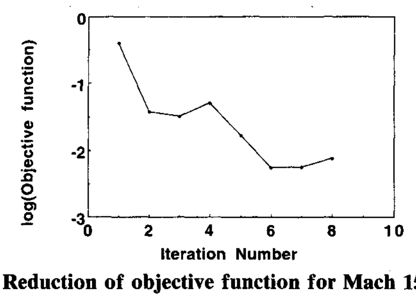

Fig. 3 Reduction of objective function for Mach 15 design.

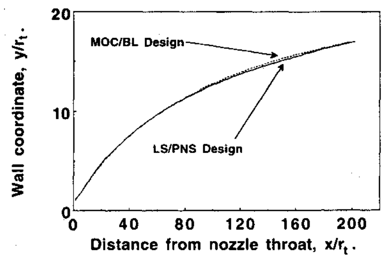

Fig. 4 Comparison of wall contours for Mach 15 design.

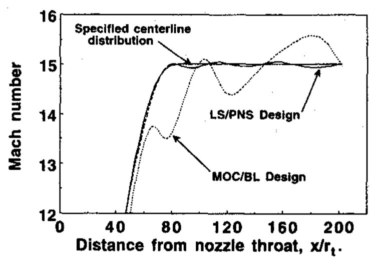

Fig. 5 Centerline Mach number distribution.

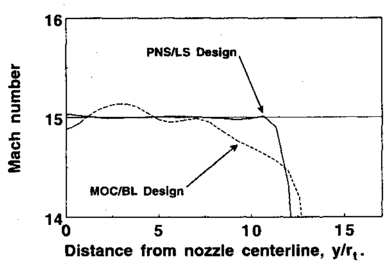

Fig. 6 Nozzle exit Mach number profiles.

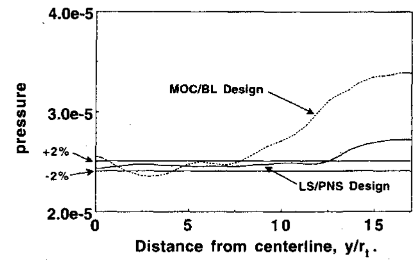

Fig. 7 Nozzle exit pressure profiles.

exactly follows the specified centerline distribution (Fig. 5). The error in Mach number should be less than ±0.06 Mach to obtain a uniform profile with less than ±2% error in static pressure. The centerline variation of Mach number for the MOC/BL design in the uniform core region is excessive, greater than ±1.0 Mach (±30% static pressure variation). The LS/PNS variation of centerline Mach number was less than ±0.06 for most of the uniform core region, with a small region almost reaching a variation of ±0.10 Mach (±3% variation in static pressure) near the beginning of the uniform core region. The LS/PNS design exit profiles for Mach number (Fig. 6), static pressure (Fig. 7, pressure nondimensionalized by the dynamic pressure at Mach 1), and flow angle (Fig. 8) show very good agreement with the design conditions and demonstrate that the flow is much better than the design goal at the nozzle exit. In addition, the uniform core at the exit extends over ~13.5 in., significantly greater than the 47% goal. The advantage of the LS/PNS design can clearly be seen in the Mach number contour plot (Fig. 9). Each contour level represents an increment of 0.2 Mach number (±3% static pressure variation). A constant uniform core flow region and a smooth expansion region are demonstrated by the LS/PNS design, which is not obtainable with the MOC/BL design.

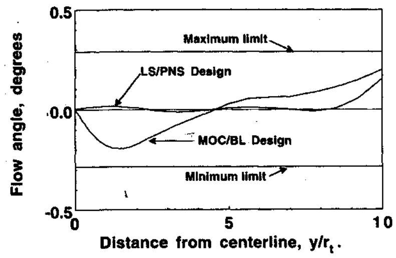

Fig. 8 Flow angle.

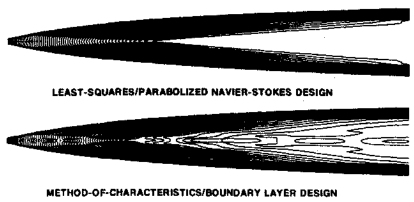

Fig. 9 Comparison of Mach number contours for LS/PNS and MOC/BL designed nozzles, 0.2 Mach increment.

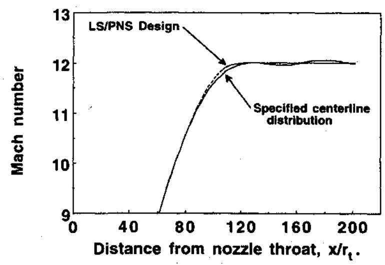

Fig. 10 Comparison of specified and computed centerline Mach number distribution for Mach 12 design.

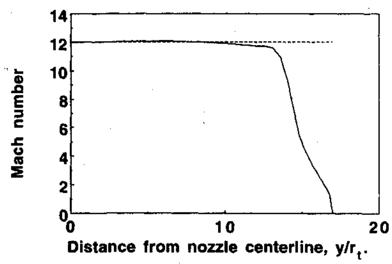

Fig. 11 Comparison of Mach exit profile with design Mach number, Mach 12 design.

These results clearly demonstrate the capability of the new procedure. However, the LS/PNS design could still be improved by using more design parameters and knots on the cubic spline, a better distribution of the location of the knots, or a different distribution and weight functions used in defining the objective function.

To demonstrate the robustness of the procedure, Mach 12 and Mach 18 nozzles were designed using the same starting contour defined by the coefficients in Table 1. This was accomplished by redefining the objective function for the expansion region and nozzle exit. For the Mach 12 case the centerline Mach number distribution was modified so that the Mach numbers at points B and C of Fig. 1 were 9 and 12, respectively. The source-flow region for the Mach 12 nozzle design was generated assuming 8-deg source flow. The centerline Mach number distribution and the exit Mach number profile are shown in Figs. 10 and 11. Excellent agreement is obtained with the design Mach numbers. The slight dropoff of Mach number shown in the Mach number profile is because the uniform core region only extends to $y/r_t = 8.3$ at this location.

For the Mach 18 case only the centerline Mach number distribution was modified, so that the Mach numbers at points

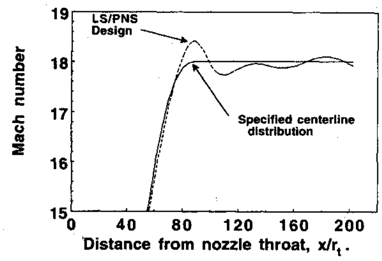

Fig. 12 Comparison of specified and computed centerline Mach number distribution for Mach 18 design.

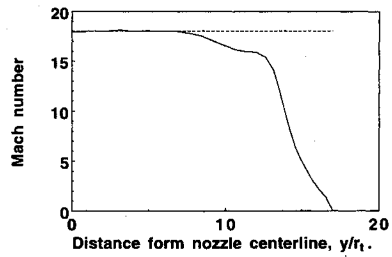

Fig. 13 Comparison of Mach exit profile with design Mach number, Mach 18 design.

B and C of Fig. 1 were 15 and 18, respectively. The centerline Mach number distribution and the exit Mach number profile are shown in Figs. 12 and 13. In this case the centerline Mach number distribution has a slight oscillation at the beginning of the uniform core region. An improved solution could be obtained by adding or moving the location of the design parameters or by decreasing the angle at which the flow is expanded in the source-flow region. The uniform core region extends to approximately $y/r_t = 7.2$ at the exit of the nozzle. The Mach number profile in this region, again, agrees with the design Mach number. It is obvious that the domain of the objective function could be extended at the exit to include more of the inviscid core. This would force a smaller throat to be used, resulting in a larger uniform core region for the same nozzle length.

Concluding Remarks

A new procedure that unifies the best of current classical design practices, CFD, and optimization procedures has been demonstrated for designing the aerodynamic lines of hypersonic wind-tunnel nozzles. An efficient CFD code, which solves the PNS equations, was coupled to an LS optimization procedure. An LS problem was formulated to minimize the difference between the computed flowfield and the objective function. The objective function consists of the centerline Mach number distribution and the exit Mach number and radial velocity profiles. The aerodynamic lines of the nozzle were defined using cubic splines, the slopes of which were optimized with the design procedure. The use of slopes at the specified control points as the design parameters improved the convergence characteristics of the design procedure and is consistent with design methods used with the MOC. The new procedure can be used to design hypersonic wind-tunnel nozzles where the classical procedure has been shown to break down. An advantage of the new procedure is that it automatically compensates the nozzle contour for viscous effects as part of the unified design procedure. The new procedure was demonstrated by designing three different helium nozzles. Mach 12, Mach 15, and Mach 18 nozzles were designed for a fixed length and exit radius. The computed flowfield for the Mach 15 LS/PNS-designed nozzles demonstrated a significant improvement over the classical MOC/BL designed nozzle. The design of the Mach 12 and Mach 18 nozzles demonstrated the robustness of the procedure by using the same starting contour as the Mach 15 design.

The new design procedure makes it practical to design high-Mach-number hypersonic wind-tunnel nozzles using CFD codes. The nozzle designer will have to specify initial and boundary conditions, centerline Mach number distribution, number and location of cubic splines, design constraints, and select a turbulence model for use in the calculations. However, once these conditions are specified, a design can be made with this procedure to satisfy most practical requirements.

Acknowledgment

The author would like to thank Anne Edwards of the Experimental Hypersonics Branch, Space Systems Division, for providing the Navier-Stokes solution used as the initial condition in the calculations.

References

[1]Erlebacher, G., Kumar, A., Anderson, E. C., Rogers, R. C., Dwoyer, D. L., Salas, M., and Harris, J. E., "A Computational Design Procedure for Actively Cooled Hypersonic Wind-Tunnel Nozzles Subject to Wall Shape Constraints," CFD in Aerospace Design Workshop, Univ. of Tennessee Space Inst., Tullahoma, TN, June 1985.

[2]Benton, J. R., "Design and Navier-Stokes Analysis of Hypersonic Wind Tunnel Nozzles," M. S. Thesis, Dept. of Mechanical and Aerospace Engineering, North Carolina State Univ., Raleigh, NC, 1989.

[3]Anderson, J. D., *Modern Compressible Flow,* 1st ed., McGraw-Hill, New York, 1982, pp. 303–305.

[4]Benton, J., Perkins, J., and Edwards, A., "Limitations of the Method of Characteristics When Applied to Axisymmetric Hypersonic Nozzle Design," AIAA Paper 90-0192, Jan. 1990.

[5]Candler, G., and Perkins, J., "Effects of Vibrational Nonequilibrium on Axisymmetric Hypersonic Nozzle Design," AIAA Paper 91-0297, Jan. 1991.

[6]Armstrong, A. H. and Smith, M. G., "Two-Dimensional Supersonic Nozzle Design, Part I: Theory," Armament Research Establishment, Rept. 5/51, London, Oct. 1951.

[7]Ruptash, J., " Supersonic Wind Tunnels—Theory, Design and Performance," Univ. of Toronto, Institute of Aerophysics, Toronto, Canada, June 1952.

[8]Sivells, J. C., "Aerodynamic Design of Axisymmetric Hypersonic Wind Tunnel Nozzles," *Journal of Spacecraft and Rockets,* Vol. 7, No. 11, 1970, pp. 1292–1299.

[9]Beckwith, I. E., and Moore, J. A., "An Accurate and Rapid Method for the Design of Supersonic Nozzles," NACA TN 3322, Feb. 1955.

[10]Johnson, C. B., "Real-Gas Effects on Hypersonic Nozzle Contours with a Method of Calculation," NASA TN D-1622, April 1963.

[11]Lukasiewicz, J., *Experimental Methods of Hypersonics,* Vol. 3, Part I, Gasdynamics, Dekker, New York, 1973, pp. 80–86.

[12]Zucrow, M. J. and Hoffman, J. D., *Gas Dynamics,* Vol. 2, Multidimensional Flow, Wiley, New York, 1977.

[13]Shapiro, A. H., *The Dynamics and Thermodynamics of Compressible Fluid Flow,* Vol. 1, Ronald Press, New York, 1953.

[14]Yang, J. Y., Lombard, C. K., Luh, R. C.-C., Nagaraj, N., and Codding, W. H., "Aerodynamic Design Modification of a Hypersonic Wind Tunnel Nozzle by CSCM with High Order Accuracy," AIAA Paper 87-1896, June 1987.

[15]Meyer, R. E., and Holt, M., "The Correction of Wind Tunnel Nozzles for Two-Dimensional, Supersonic Flow," *The Aeronautical Quarterly,* Vol. 2, Nov. 1950, pp. 195–208.

[16]Gielda, T., and McRae, D., "An Accurate, Stable, Explicit, Parabolized Navier-Stokes Solver for High Speed Flows," AIAA Paper 86-1116, May 1986.

[17]Korte, J. J., "An Explicit Upwind Algorithm for Solving the Parabolized Navier-Stokes Equations," NASA TP 3050, Feb. 1991.

[18]Huddleston, D. H., "Aerodynamic Design Optimization Using Computational Fluid Dynamics," Ph.D. Dissertation, Univ. of Tennessee, Knoxville, TN, 1989.

[19]Korte, J. J., Kumar, A., Singh, D. J., and Grossman, B., "Least Squares/Parabolized Navier-Stokes Procedure for Optimizing Hypersonic Wind-Tunnel Nozzles," AIAA Paper 91-2273, June 1991.

[20]Scales, L. E., *Introduction to Non-Linear Optimization,* Macmillan, London, 1985, pp. 110–136.

[21]Kumar, A., "Numerical Analysis of the Scramjet-Inlet Flow Field by Using Two-Dimensional Navier-Stokes Equations," NASA TP-1940, May 1981.

[22]Baldwin, B. S., and Lomax, H., "Thin Layer Approximation and Algebraic Model for Separated Turbulent Flows," AIAA Paper 78-0257, Jan. 1978.

Ernest V. Zoby
Associate Editor